Quantitative Analysis on the Tonal Quality of Various Pianos

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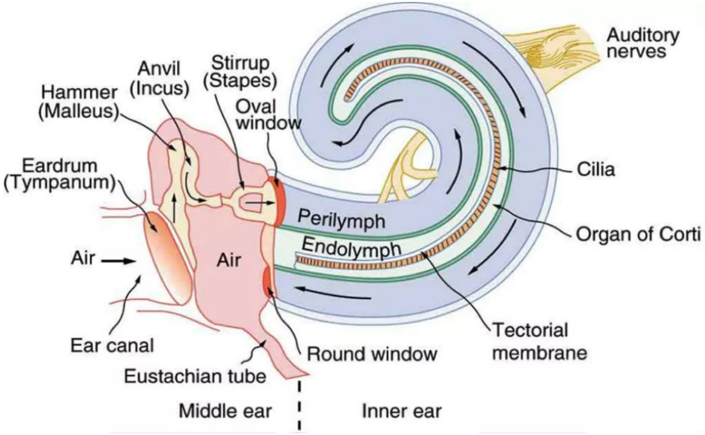
PHYS 398 DLP - University of Illinois at Urbana-Champaign
26 April 2019

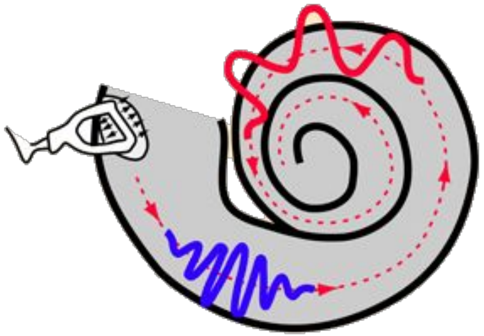
Presentation Outline

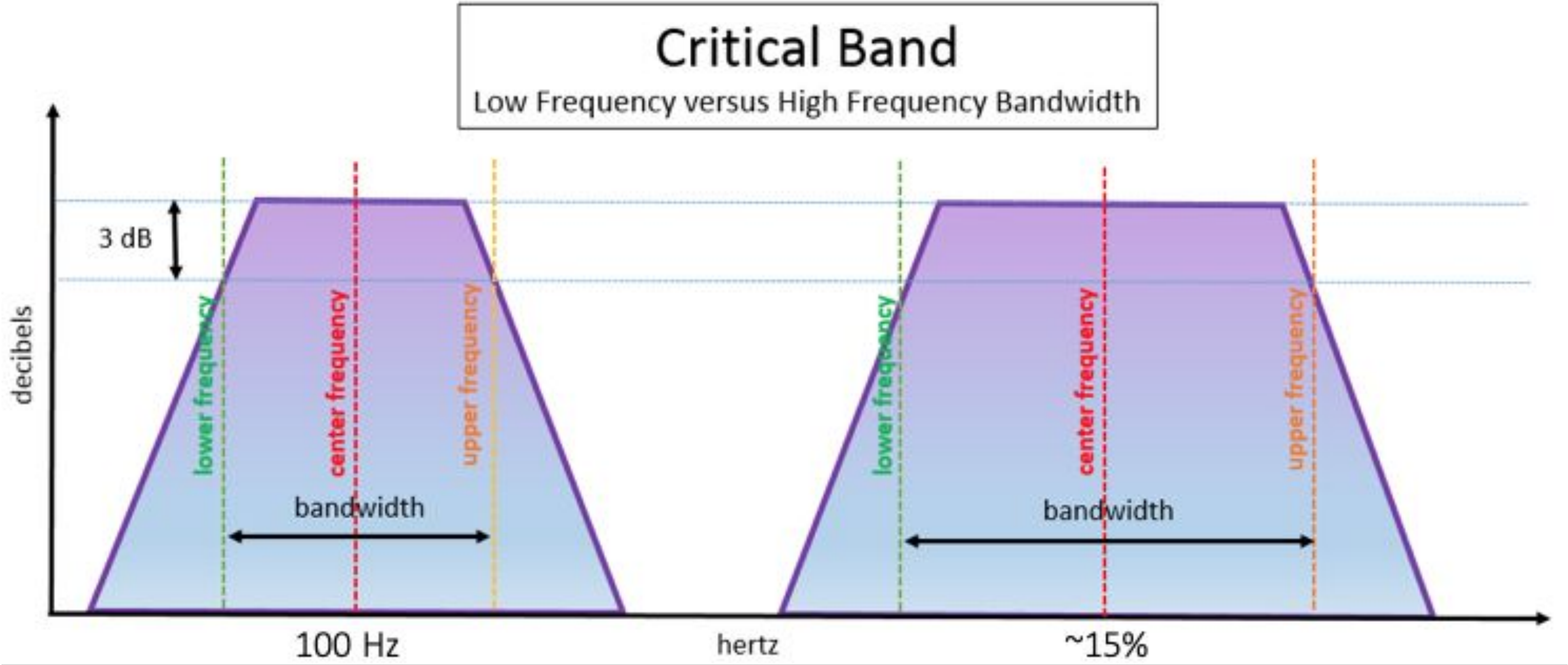
- Theory
    - Auditory perception
    - Tuning methods
    - Inharmonicity
- Project Goals
- Methods
    - PCB construction
    - Recordings
    - Analysis
- 4. Results
    - Frequency shifts
    - Octave correspondence
    - Overtone amplitude
    - Self-dissonance
- 5. Conclusion
- 6. Discussion

Theory - What makes a chord sound good?

- Inner ear anatomy
  - Cochlear duct is a series of fluid-filled chambers responsible for auditory perception
  - Organ of Corti transforms pressure waves (sound) to electrical nerve signals using cilia
    - Different frequencies excite different regions of cilia → **critical bands**






Critical Band
Low Frequency versus High Frequency Bandwidth
3 dB
decibels
lower frequency
center frequency
upper frequency
bandwidth
lower frequency
center frequency
upper frequency
bandwidth
100 Hz
hertz
~15%

Theory - Equal temperament

- 12-tone equal temperament adopted in Western classical music for convenience with modern piano design and minimized dissonance
  - Other tuning methods can minimize dissonance in certain intervals but would result in increased dissonance in most other intervals
  - Equal temperament spreads this dissonance across entire piano

$$f_n = f_a(\sqrt[12]{2})^{(n-a)}$$

- Frequencies of successive notes separated by constant multiplicative factor of

$$\sqrt[12]{2} \approx 1.059463$$

Musical Note	Frequency (Hz)	f_{Note}/f_{A_3}
A_3	220.0	1
A sharp/B flat	233.1	$2^{1/12}$
B_3	246.9	$2^{2/12}$
C_4	261.6	$2^{3/12}$
C sharp/D flat	277.2	$2^{4/12}$
D_4	293.7	$2^{5/12}$
D sharp/E flat	311.1	$2^{6/12}$
E_4	329.6	$2^{7/12}$
F_4	349.2	$2^{8/12}$
F sharp/G flat	370.0	$2^{9/12}$
G_4	392.0	$2^{10/12}$
A sharp/B flat	415.3	$2^{11/12}$
A_4	440.0	$2^{12/12}$

- A "pure" tone is characterized by a sine wave oscillating at a single frequency
  - Determining consonance and dissonance between two pure tones is as simple as comparing two frequencies
- Pianos produce "complex" tones comprised of many frequencies (harmonics)
  - Determining consonance and dissonance becomes more complicated

$$f_n = nf_0$$

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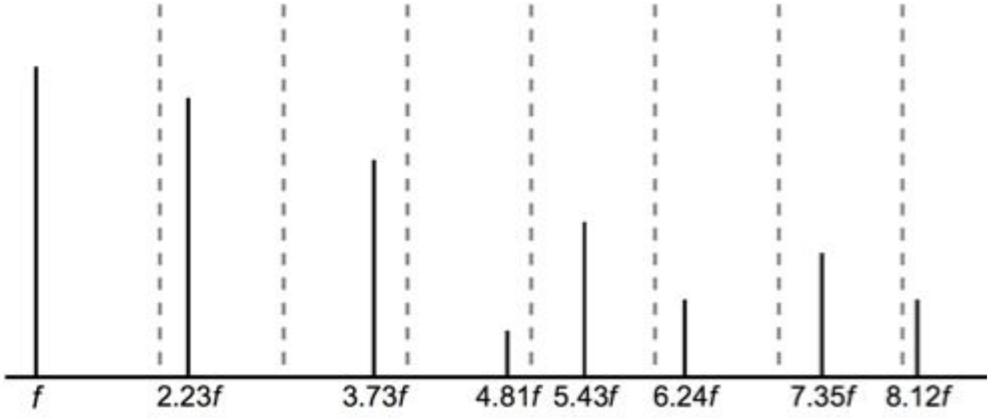
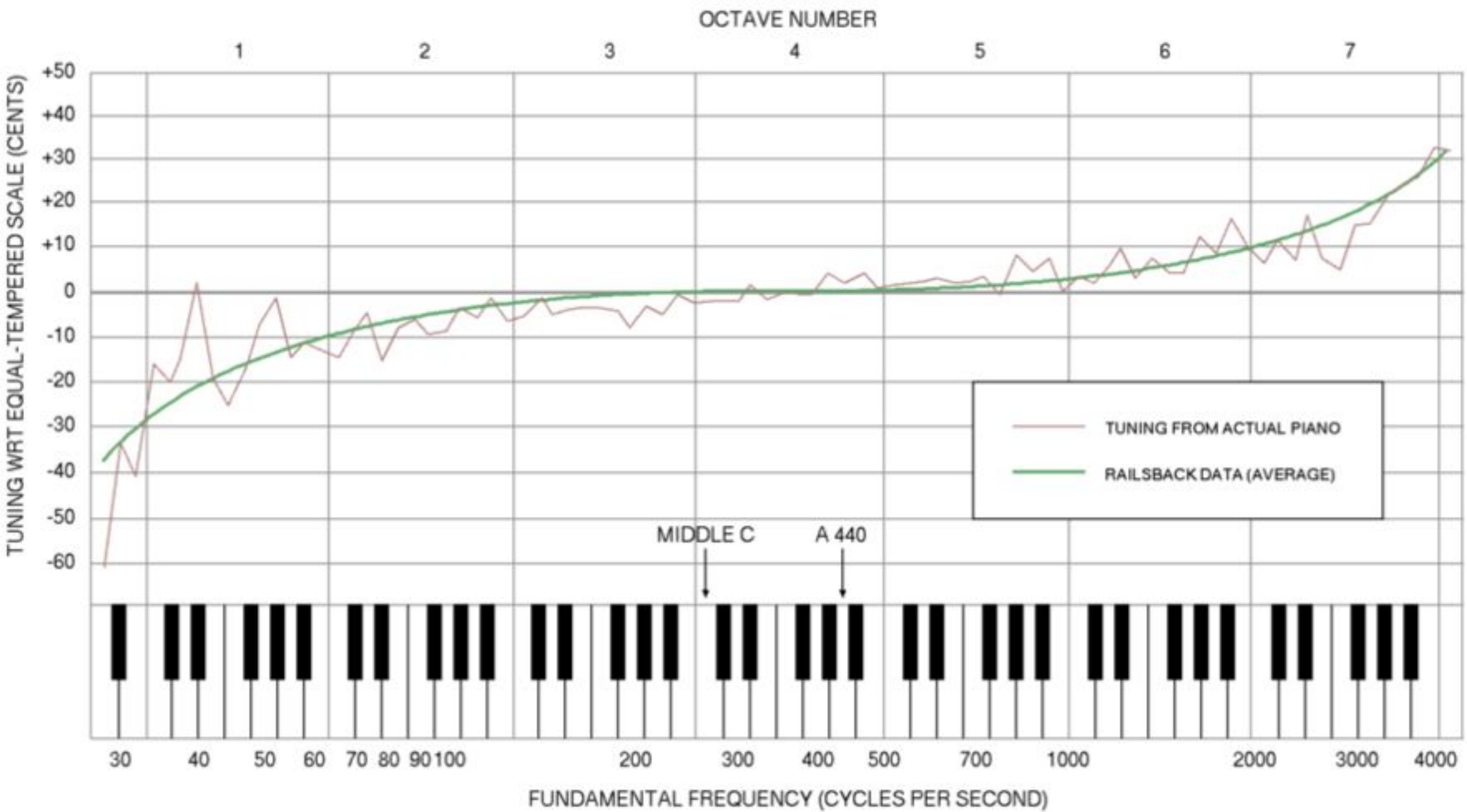
Note	1st	2nd	3rd	4th
C_4	261.6 Hz	523.2 Hz	784.8 Hz	1046 Hz
C_5	523.2 Hz	1046 Hz	1570 Hz	2093 Hz

Note	1st	2nd	3rd	4th
F_4	349.2 Hz	698.4 Hz	1048 Hz	1397 Hz
F sharp	370.0 Hz	740.0 Hz	1110 Hz	1480 Hz

Theory - Inharmonicity

- The frequencies of harmonics begin to drift from integer multiples of the fundamental
    - Rigidity of piano does not propagate sound waves efficiently (acoustical impedance)
- Amount of inharmonicity is dependent on instrument/string characteristics (tension, stiffness, length)
- More elasticity = less inharmonicity



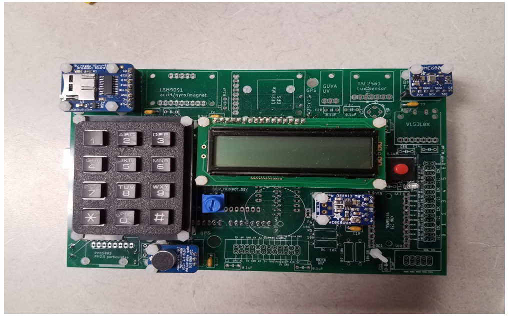

OCTAVE NUMBER
1
2
3
4
5
6
7
TUNING WRT EQUAL-TEMPERED SCALE (CENTS)
+50
+40
+30
+20
+10
0
-10
-20
-30
-40
-50
-60
TUNING FROM ACTUAL PIANO
RAILSBACK DATA (AVERAGE)
MIDDLE C
A 440
30
40
50
60
70
80
90
100
200
300
400
500
700
1000
2000
3000
4000
FUNDAMENTAL FREQUENCY (CYCLES PER SECOND)

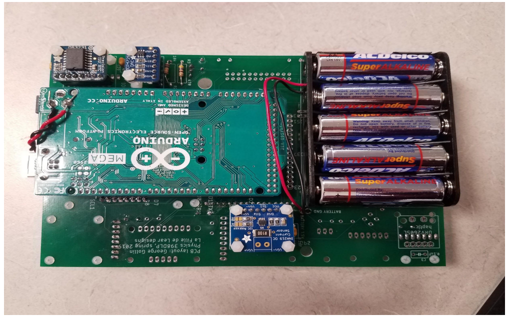
Project Goals

1. Quantitatively determine the differences between a tuned and an untuned piano
2. Determine the effect of frequency shift, octave correspondence, overtone amplitude, and self-dissonance on the tonal quality of a piano

Methods

- Hardware
  - PCB
    - Arduino microcontroller
  - Sensors
    - Electret microphone
    - LCD
    - Keypad
    - Current sensor
    - Mono amplifier
    - RTC
    - BME 680
    - SD breakout



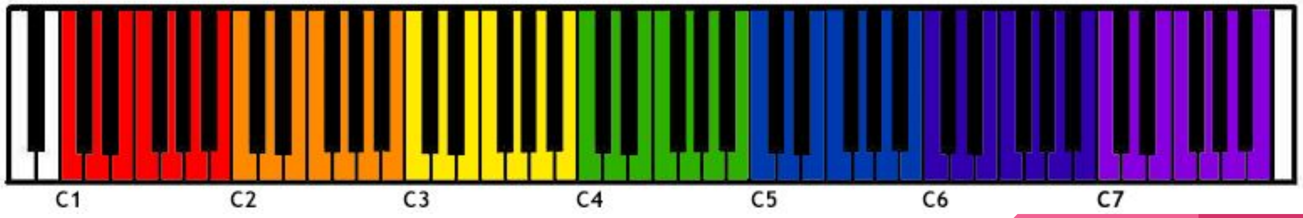


Methods Continued

- Types of recordings
  - Tuned and untuned
    - Steinway
      - Grand
    - Yamaha
      - Upright and grand
    - Mason & Hamlin
      - Grand
  - Recently tuned and not recently tuned
  - Krannert Center for Performing Arts

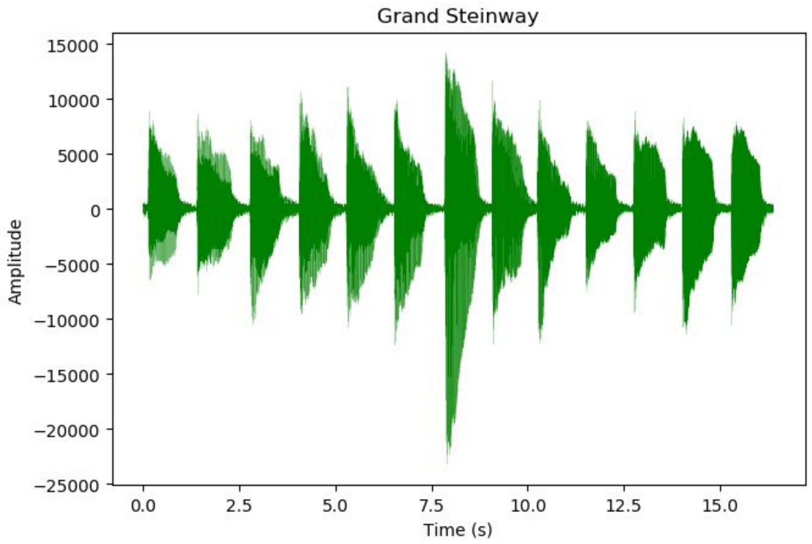
Methods Continued

- Recording procedure
  - Originally every key and middle C (C4)
    - Pedals: sustain, damper, staccato
    - Similar information from subsequent octaves
  - Changed to octaves C2, C4, and C5 and middle C
    - Orange, green, indigo
    - Black and white
    - Only analyzed white keys
    - Allowed time between notes

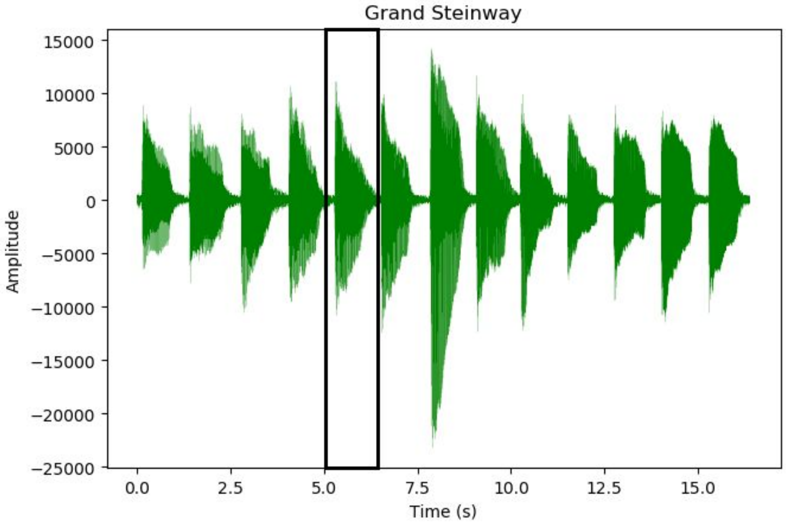


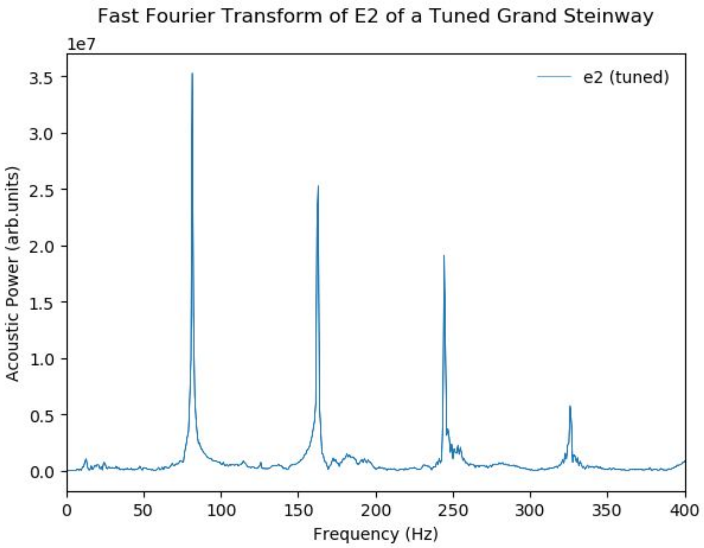
Methods Continued

- Offline analysis
  - Python
  - Arduino to SD as binary
  - Binary to wave
    - Gollin’s code
  - Graph wave file
    - Amplitude vs. time
  - Duration of each note
  - Cut file for each note
    - Numpy FFT
      - Forward Discrete Fourier Transform
        - Acoustic power coefficient
      - Computes frequencies corresponding to coefficients

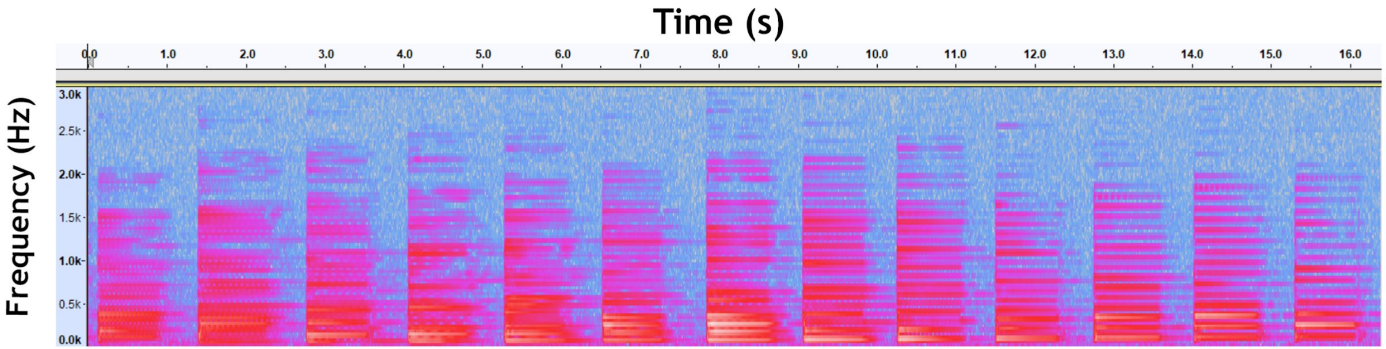


Methods Continued





Methods Continued



Spectrogram

- C2 Scale, tuned Steinway
- Data transformed from time domain to frequency domain
  - Fourier transform
- Vertical line
  - Notes
- Color - intensity

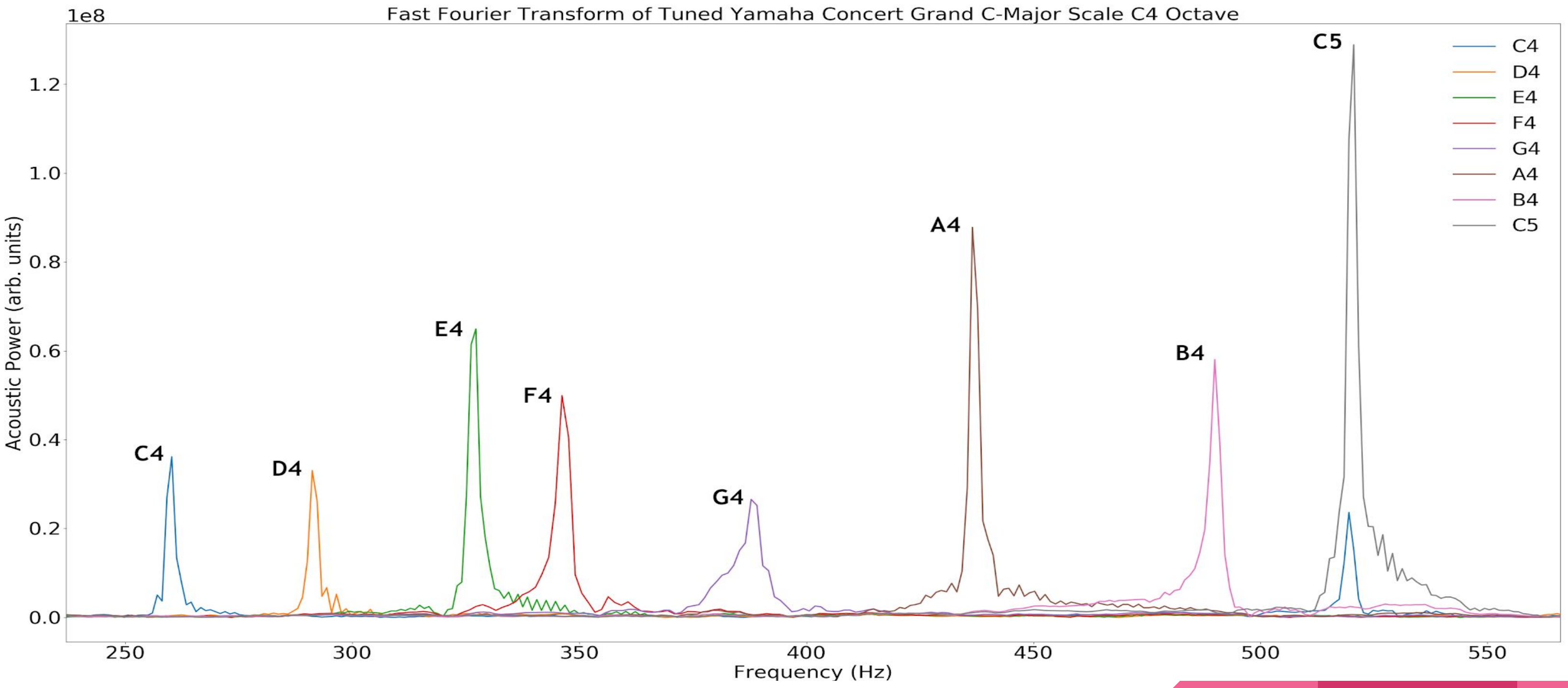
Results

- General FFT
- Frequency shifts
- Octave correspondence
- Overtone amplitude
- Self-dissonance

Fast Fourier Transform

- As mentioned before, a FFT brings the audio file from the time domain into the frequency domain
- Using a FFT will produce frequency peaks where the fundamental pitch resides
- The tonal quality of a piano can be analyzed by using the difference between the measured and theoretical fundamental frequency

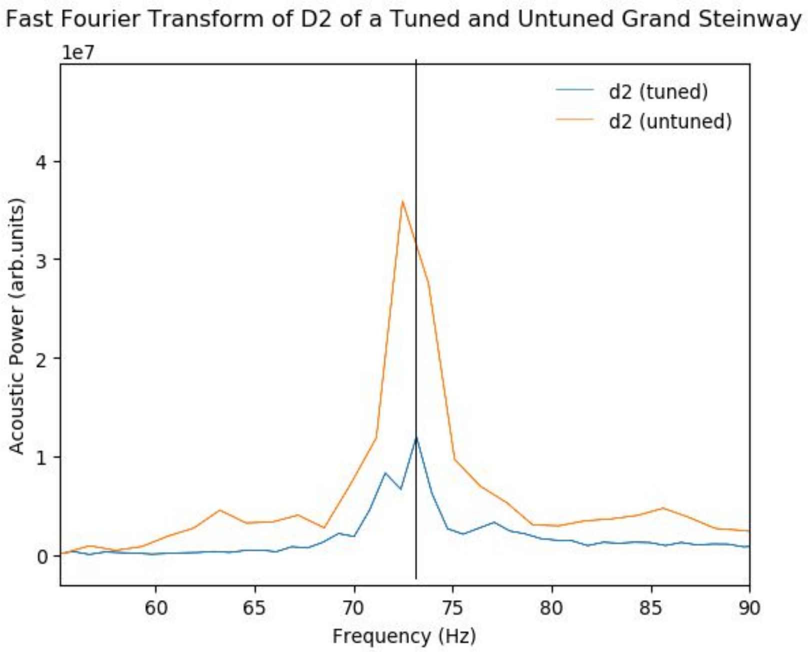
Fast Fourier Transform on C-Major Scale

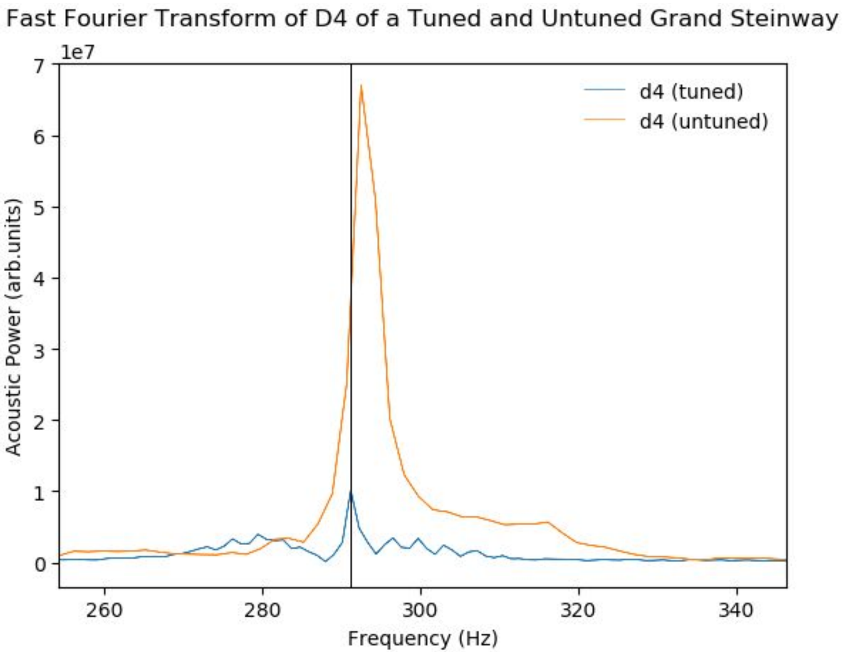


Frequency Shifts

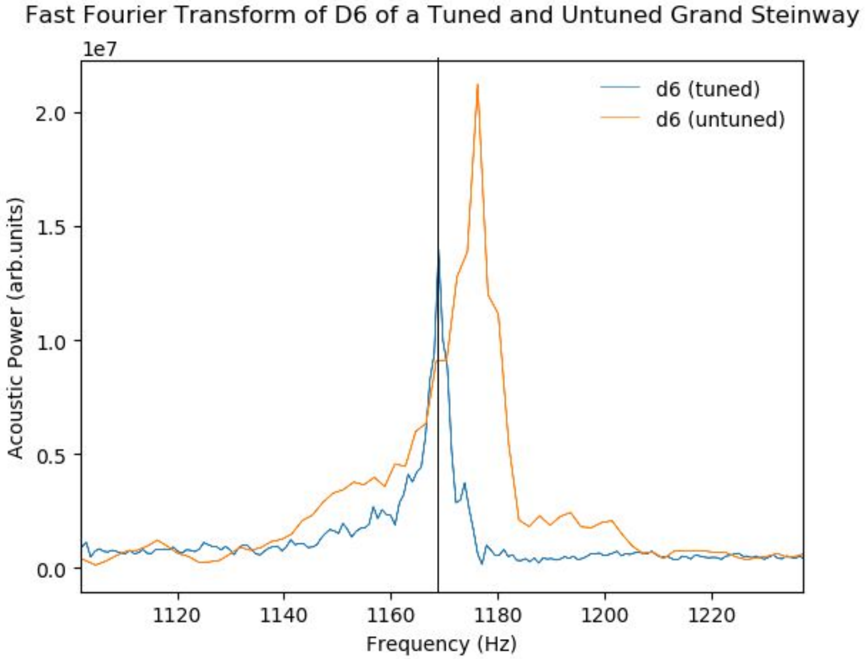
- They are the largest contributor to impurities in tonal quality.
- When the frequency of a note deviates noticeably from its equal tempered frequency, it is perceived as sharp or flat
  - A frequency above the fundamental is sharp
  - A frequency below the fundamental is flat

Frequency Shifts Cont.





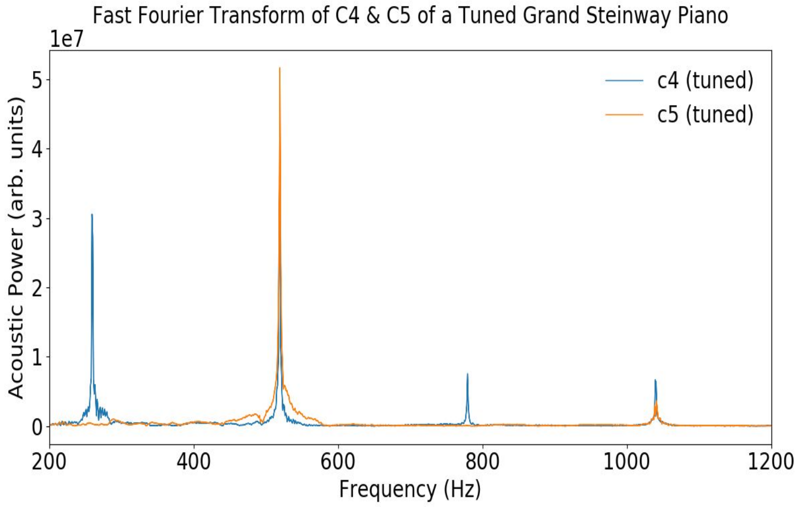
Frequency Shifts Cont.

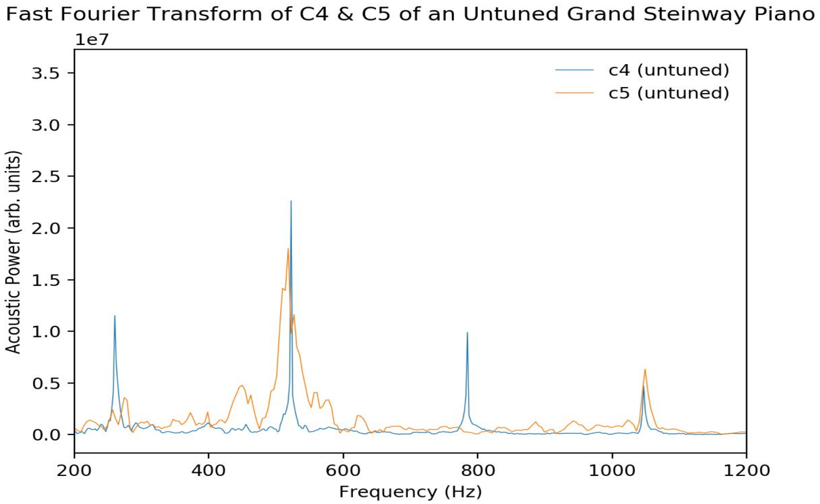


Note	Grand Steinway (Tuned) 1st Fundamental (Hz)	Grand Steinway (Untuned) 1st Fundamental (Hz)	Abs. Diff(Hz)
C2	64.3134	64.7931	0.4797
D2	72.6644	72.4679	0.1965
E2	81.4966	81.9685	0.4719
F2	86.0679	86.5395	0.4716
G2	96.5554	97.1446	0.5892
A2	108.647	110.227	1.58
B2	122.037	123.212	1.175
C5	519.385	518.113	1.272
C6	1042.58	1048.58	6
D6	1168.99	1176.3	7.31
E6	1313.78	1322.94	9.16
F6	1393.56	1400.1	6.54
G6	1564.4	1576.34	11.94
A6	1758.03	1770.13	12.1
B6	1973.52	1990.78	17.26

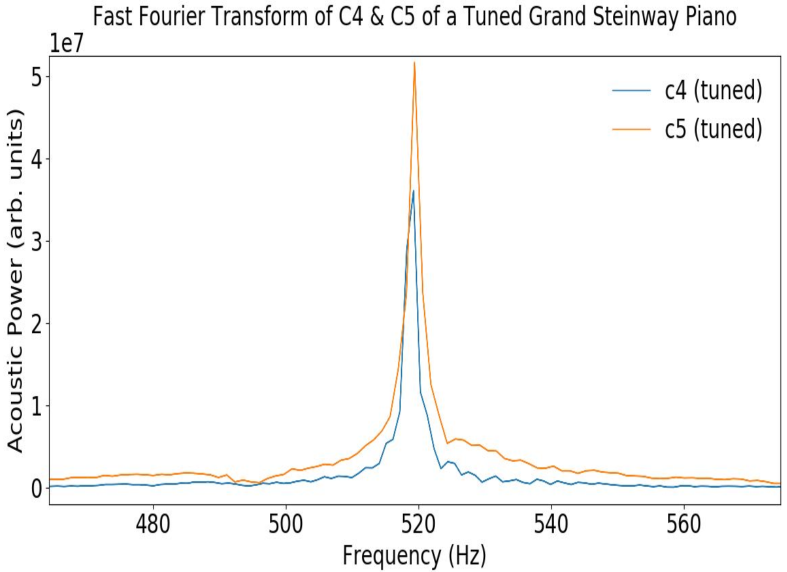
Octave Correspondence

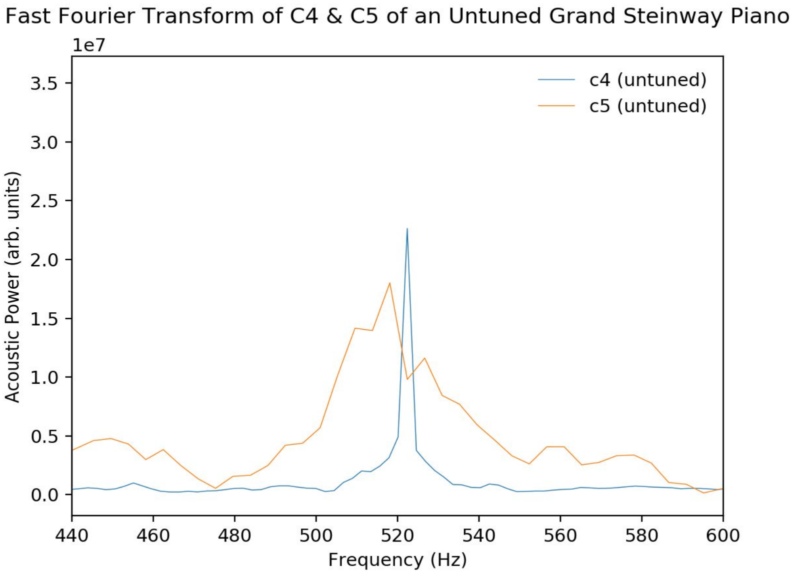
- Primary method used to tune pianos
  - Align the second harmonic of C4 with first fundamental of C5





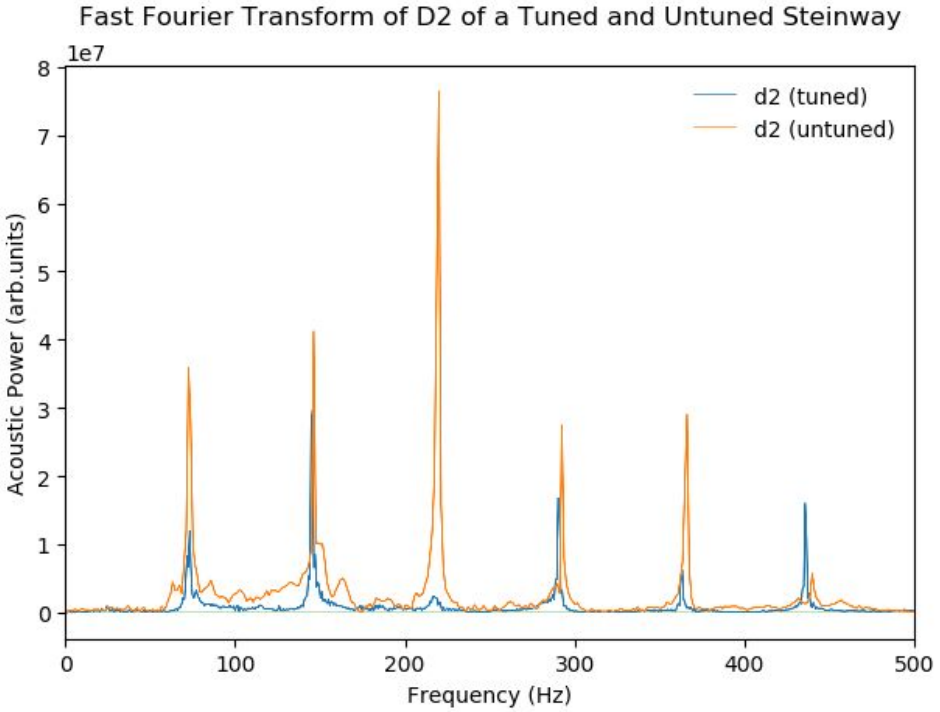
Octave Correspondence Cont.





Overtone Amplitude

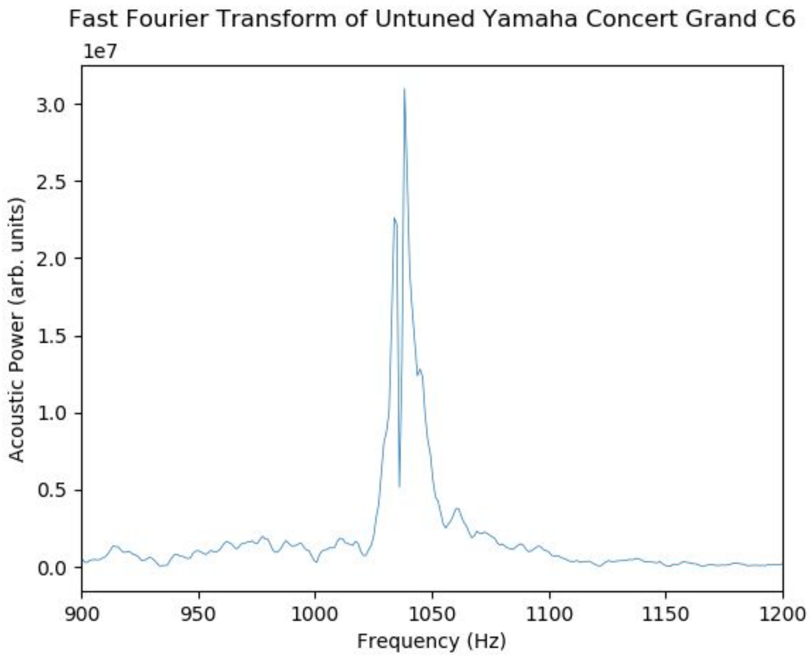
- The perceived frequency and tone of a note is due to the prevalence of its harmonic.
- When the acoustic power of a note's upper harmonics begin to exceed that of its fundamental, the frequency of the fundamental begins to get overpowered.

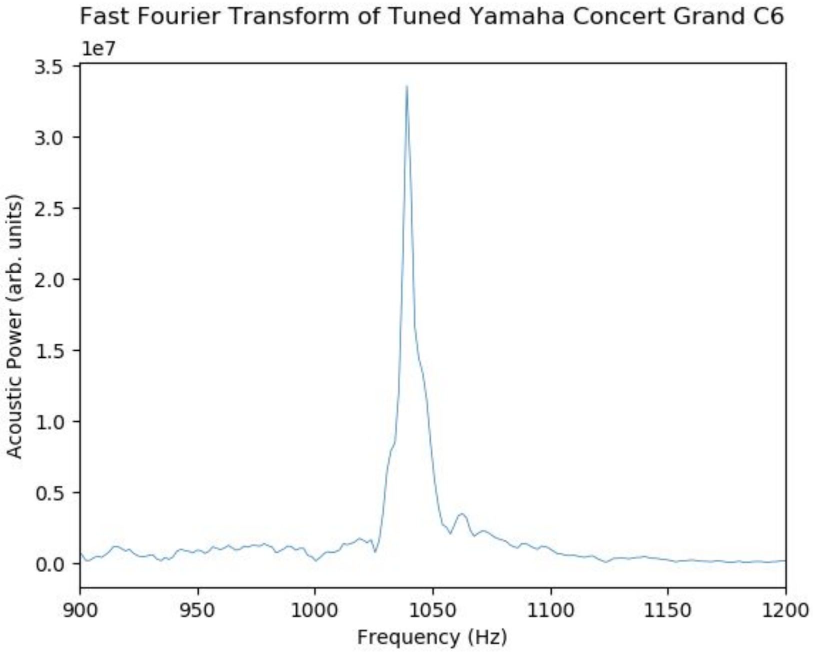


Self-Dissonance

- When a piano is out of tune, a listener can often hear beats when it’s played
  - Two or more tones of similar frequencies interfering with each other
- An untuned piano can display doublet shaped peaks, whereas a tuned piano has a single peak
- Doublet shape is caused by dissonance.
  - Cannot form in lower octaves (one string per note)
  - Middle and upper octaves have multiple strings per note

Self-Dissonance





Discussion

- Sources of error
- Adjustments for future experiments
- Design proposal

Sources of Error

- Not all results are standardized across all four devices
- FFT peak values were determined manually
- More tuned than untuned pianos were recorded

Future Improvements

- Automating the code to generate the FFT peak value
- A higher quality microphone could be used
- Recording barometric pressure, temperature, and humidity may be useful
- Focus on a single piano for an extended period of time

Design Proposal

- This analysis can be used for a variety of piano technician needs
  - Piano appraisal
  - Training piano tuners
  - Verifying tonal quality before concerts
- The methods used in this paper can be used to create a software for personal use
  - Takes a scale as an input
  - Eliminates white noise
  - Analyzes FFT
  - Generates and compares Railsback curve

Conclusion

- Perceived tonal quality doesn’t entirely depend on frequency shifts
- Tuned pianos exhibit small frequency differences, strong octave correspondence, smooth overtone amplitude patterns, and low self dissonance
- Untuned pianos exhibit large frequency differences, poor octave correspondence, erratic overtone amplitude patterns, and noticeable self-dissonance

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